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Analysis of seasonal cycles in climatic trends with application to satellite observations of sea ice extent

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[1] We present a new technique to study the seasonal cycle of climatic trends in the expected value, variance, and other moments of the statistical distribution. The basis of the technique is multiple linear regression, but with periodic basis functions. The technique allows us to provide comprehensive information on statistical parameters of climate for every day of an observational period. Using daily data, the technique has no problems caused by different lengths of months or the leap-year cycle. Without needing to assume the stationarity of contemporary climate, the technique allows the study of statistical parameters of climatic records of arbitrary length. We illustrate the technique with applications to trends in the satellite observed variations of sea ice extent in the Northern and Southern Hemispheres. We show that a significant part of the variability in hemispheric sea ice extents for the period 1978–1999 is related to linear trends. *INDEX TERMS:* 1620 Global Change: Climate dynamics (3309); 1866 Hydrology: Soil moisture; 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 4215 Oceanography: General: Climate and interannual variability (3309)

1. Introduction

[2] Traditional climatology uses monthly averages and statistics estimated for 30-yr time intervals. This approach, however, has well-known limitations. Monthly averages incorporate different lengths for different months and different years (leap years). Using only 12 monthly averages is a very crude approximation of the seasonal variation for many meteorological variables and produces biases in the spectrum of meteorological records. Time series for specific months are too short to estimate the parameters of the statistical distribution of monthly averages properly.

[3] Here we introduce a new statistical approach to analyzing climate data that explicitly accepts the idea that long-term climatic records may contain trends in means, variances, covariances, and other moments of the statistical distribution of meteorological variables. We also explicitly permit seasonal cycles of these trends. This approach is designed to analyze observed past climate change and climate model output, but is not designed to be used to extrapolate the trends into the future.

2. Theory

[4] We start by using daily observations instead of monthly averages. Let us denote

$$y(t) = Y(t) + y'(t); \tag{1}$$

where $y(t)$ is the observed value of y for day number t, $t = t_1, t_2, t_3, \ldots, t_n$; $Y(t)$ is the expected value of $y(t)$; and $y'(t)$ is the residual (anomaly). We can use as t the Julian day number, or choose any other specific date as the reference day ($t = 0$).

[5] A common approach in studies of the seasonal cycle is to use Fourier harmonics of the annual period to approximate the seasonal variation of the mean values of the observed data [*Anderson*, 1971; *Polyak*, 1975]. It is well known that climatic trends may be different for different seasons [*Mitchell*, 1961; *Vinnikov*, 1986; *Hansen and Lebedeff*, 1987; *Chapman and Walsh*, 1993; *Stammerjohn and Smith*, 1997; *Hansen et al.*, 1999; *Parkinson et al.*, 1999; *Jones et al.*, 1999; *Rigor et al.*, 2000; *Serreze et al.*, 2000]. Let us assume that the climatic trend in the expectation $Y(t)$ of the observed variable $y(t)$ is a periodic function of the annual period. Particularly, for a linear trend this means:

$$Y(t) = A(t) + B(t) \cdot t, \tag{2}$$

where $A(t) = A(t + T)$, $B(t) = B(t + T)$, and $T = 365.25$ days. Let us use a limited number of Fourier harmonics to approximate both periodic functions, $A(t)$ and $B(t)$:

$$\begin{aligned} Y(t) = a_0 + \sum_{k=1}^{K} \left[a_k \sin\left(\frac{2\pi kt}{T}\right) + b_k \cos\left(\frac{2\pi kt}{T}\right) \right] \\ + \alpha_0 t + \sum_{m=1}^{M} \left[\alpha_m t \sin\left(\frac{2\pi mt}{T}\right) + \beta_m t \cos\left(\frac{2\pi mt}{T}\right) \right]. \end{aligned} \tag{3}$$

We can estimate all the unknown multiple regression coefficients in (3) from the least squares condition:

$$\begin{aligned} \sum_{t=t_1}^{t_n} [y(t) - Y(t)]^2 \\ = F(a_0, \ldots, a_K, b_1, \ldots, b_K, \alpha_0, \ldots, \alpha_M, \beta_1, \ldots, \beta_M) = \text{min}. \end{aligned} \tag{4}$$

The number of harmonics required for approximation of $A(t)$ and $B(t)$, K, and M, should be chosen from independent considerations or should be estimated from analyses of the same data.

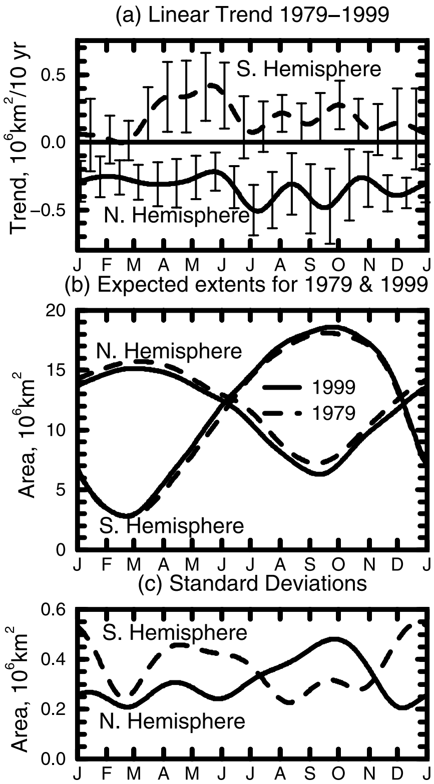

Figure 3. Sea ice extents in the Northern and Southern Hemispheres: (a) linear trend 1979–1999, (b) expected extent for years 1979 and 1999, (c) standard deviations of daily extents, 1979–1999.

4. Discussion and Conclusions

[10] Most previous analyses of trends in sea ice extents have produced statistics for monthly or yearly averages. Daily observations give us an opportunity to see additional interesting details in the same climatic records. Results of applying the new technique to sea ice extents yield the following conclusions:

4.1. Northern Hemisphere Sea Ice Extent

[11] The expected sea ice extent in the Northern Hemisphere has been decreasing during 1979–1999 for each day of the year with rates from -0.2×10^6 km^2/10 yr to -0.5×10^6 km^2/10 yr (Figures 3a and 3b). Consistently, although in less detail, *Parkinson and Cavalieri* [2002] find that monthly trends in the Northern Hemisphere ice extents have negative values for all 12 months, all with magnitudes of at least -0.23×10^6 km^2/10 yr.

[12] The expected maximum seasonal sea ice extent in 1999 was about 0.5×10^6 km^2 less than it was in 1979 (Figure 1, line (a)). This corresponds to a $\sim$0.3° latitude poleward shift of the average position of the sea ice boundary at the time of its maximum seasonal expansion.

[13] The expected minimum seasonal sea ice extent in 1999 was about 1.0×10^6 km^2 less than it was in 1979 (Figure 1, line (b)). This corresponds to a $\sim$1° latitude poleward shift of the average position of the sea ice boundary at the time of its maximum seasonal retreat.

[14] The average amplitude of the seasonal variation in sea ice extent has increased about 0.5×10^6 km^2 during the 21-year period 1979–1999, from $\sim 8.5 \times 10^6$ km^2 to $\sim 9.0 \times 10^6$ km^2 (Figure 1).

[15] Daily anomalies of sea ice extent (Figure 1) are an order of magnitude smaller than the seasonal variation of sea ice extent (about 8.5–9.0 $\times 10^6$ km^2/10 yr, Figure 1).

[16] Trend-related changes during the last two decades of the 20th Century have the same order of magnitude ($0-1 \times 10^6$ km^2) as daily anomalies of sea ice extent (Figure 1).

4.2. Southern Hemisphere Sea Ice Extent

[17] The expected sea ice extent in the Southern Hemisphere has been increasing during 1979–1999 for almost every day of the year, with rates up to $\sim 0.4 \times 10^6$ km^2/10 yr (Figures 3a and 3b). This asymmetry between Northern and Southern Hemisphere sea ice variation was predicted using a climate model forced by increasing greenhouse gases [*Manabe et al.*, 1992] and then observed through satellite microwave measurements [*Cavalieri et al.*, 1997].

[18] The expected seasonally maximum sea ice extent in 1999 is about 0.5×10^6 km^2 larger than it was in 1979 (Figure 2, line (a)). There is no noticeable trend in sea ice extent at the time of its maximum seasonal retreat (Figure 2, line (b)). The average amplitude of seasonal variation in sea ice extent has increased about 0.5×10^6 km^2 during the 21-year period, from $\sim 15.3 \times 10^6$ km^2 to $\sim 15.8 \times 10^6$ km^2. Thus the seasonal amplitude of sea ice extent has increased in both hemispheres, and in both cases by about 0.5×10^6 km^2 (Figures 1 and 2).

[19] Daily anomalies of sea ice extent in the Southern Hemisphere (Figure 2) are an order of magnitude smaller than the seasonal variation of sea ice extent (Figure 3b), the same as in the Northern Hemisphere.

[20] As in the Northern Hemisphere, daily anomalies in the Southern Hemisphere are generally between -1×10^6 km^2 and 1×10^6 km^2 (Figures 1 and 2), although the 1979–1999 trends are generally smaller in magnitude, as well as being opposite in sign (Figure 3a).

4.3. Further Development of the Technique

[21] Harmonic functions may not be optimal for approximation of seasonal variations of many climatic variables. Other classes of periodic functions may be used, and empirical statistically orthogonal functions should be considered as possible alternatives to harmonic functions.

[22] In some cases, significant seasonal variations can exist in the variance of climatic indices. In such cases, it might be useful to repeat the calculations using a "weighted least squares" with weights that are inversely proportional to the estimated variance.

[23] Trend estimates for each day of a year provide information about the seasonality of trends in moments of the statistical distribution of climatic variables. The same technique can be applied to analyze time series of observed climatic indices at every specific time of a day. In the case when we have hourly meteorological observations, we can put together the estimates for every hour and receive a full picture of seasonal and diurnal cycles in climatic trends. Joint analysis of the seasonal and diurnal cycles in climatic trends will be discussed in another paper.

[24] The existing traditional practice of climatic services is based on monthly averages and a 30-year period for normals. Even for a variable with an autocorrelation time scale of 50 days, such as sea ice, this technique provides high temporal resolution depictions of the seasonal cycles of the means, anomalies, and trends (Figures 1–3). Without needing to assume the stationarity of contemporary climate, our approach allows the study of statistical parameters of climatic records of arbitrary length. This opens a new opportunity to modernize existing climate services.